BBC RD 1977/16

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1974
THE QUEEN'S AWARD
TO INDUSTRY

REPORT

M.F. lattice planning on a spherical earth

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THE BRITISH BROADCASTING CORPORATION April 1977

BBC RD 1977/16
UDC 621.396.029.53
528.3

MF LATTICE PLANNING ON A SPHERICAL EARTH

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Summary

Planning of frequency assignments using a lattice network on a plane surface offers a uniform number of assignments per unit area and uniform levels of interference for each transmitter. A study of the problem of fitting a lattice network to the spherical surface of the Earth, without affecting these basic properties, shows that the representation of the Earth by an icosahedron is not a satisfactory solution and that the use of equal-area map projections is to be preferred.

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April 1977

(RA-157)

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Section	Title	Page
	Summary	Title Page
1.	Introduction	1
2.	Representation of a sphere by a polyhedron	1
3.	Use of equal-area map projections	2
4.	Conclusions	2
5.	References	2
	APPENDIX: Projection of lattice networks to a sphere from a tetrahedron, octahedron or icosahedron	7

MF LATTICE PLANNING ON A SPHERICAL EARTH

P.A. Laven

1. Introduction

Planning of frequency assignments using an infinite linear lattice network on a plane surface offers two basic properties:

(i) the levels of interference are uniform for each assignment

(ii) the number of assignments per unit area (the assignment density) is uniform.

If the lattice network is finite, interference levels are lower for assignments near the edge of the network than for assignments in the centre of the network. If the surface is spherical, the plane lattice network must be distorted to fit the spherical surface. This process can result in non-uniform interference levels or assignment densities.

The variations in interference levels due to the finite nature of the lattice network as a result of the limited land masses of the Earth have to be accepted, but it is desirable that the process of fitting the lattice network to the sphere should not cause any further significant variations in interference levels or assignment densities.

Two methods of projecting plane lattice networks on to a spherical surface will be considered. Eden and Minne[1] suggested that the sphere could be represented by a polyhedron. It is shown that the resulting lattice networks have non-uniform interference levels and assignment densities. An alternative method using equal-area map projections is described and it is shown that this method retains the basic properties of lattice networks on a plane surface.

This investigation was conducted in preparation for the LF/MF Regional Administrative Broadcasting Conference (1974/75) but the first session of the conference decided not to adopt lattice planning methods.

2. Representation of a sphere by a polyhedron

The sphere may be approximated by a polyhedron which has a number of plane surfaces. For use in lattice planning, it is convenient if the plane surfaces of the polyhedron are equilateral triangles, so that the equilateral triangles of the plane lattice network fit the surfaces of the polyhedron in a regular manner. There are only three solids which satisfy this criterion: they are the tetrahedron (four sides), octahedron (eight sides) and icosahedron (twenty sides).

The equilateral plane triangles of these solids correspond to equilateral triangles on the surface of the sphere. If the Earth is represented by these solids, the lengths of the sides of these equilateral spherical triangles are about 12,000 km, 10,000 km, and 7,000 km for the tetrahedron, octahedron and icosahedron respectively. The plane lattice network is superimposed on the plane surfaces of the polyhedron and is projected on to the surface of the sphere using a gnomonic (or central) projection.

The tetrahedron and octahedron are extremely poor approximations to a sphere and hence are not considered further in this section.

Fig. 1 shows a lattice network which has been generated using an icosahedron. The blue triangles correspond to the faces of the icosahedron. The red triangles correspond to the equilateral triangles of the plane lattice network and indicate the position of co-channel transmitters. These triangles vary in size and shape on the surface of the sphere because of distortion caused by the projection. The distances between transmitters tends to be less near the vertices of the icosahedron. The average co-channel distance is about 2,900 km, but it is obvious from Fig. 1 that there are considerable departures from this nominal distance. For each transmitter in the network, the multiple interference caused by all transmitters in the network has been calculated by two methods:*

a) the 'spherical earth' method – the distance between transmitters is the great circle distance calculated from the geographical co-ordinates of the transmitters

b) the 'plane earth' method – the distance between transmitters is calculated using the plane lattice network and the nominal co-channel distance.

Method (a) demonstrates the actual limitations when using the projected lattice while (b) demonstrates the limitations that arise solely from the finite area covered by the lattice. The results are shown in Fig. 1. There are a number of assignments for which interference levels are significantly different for the spherical and plane lattices. The greatest discrepancies occur when the great circle path between two assignments traverses parts of three faces of the icosahedron, such as between the assignments at 60·5°N 115·4°E and 40·9°N 120·5°E.

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* Calculations of usable field strength are based on a network of medium-frequency transmitters with an e.m.r.p. of 1 kW and a co-channel protection ratio of 30 dB. The assumed relationship between field strength E in dBμ and distance D in km is given by

$$E = \frac{396}{4 + 0{\cdot}001\,D} - 40$$

It is not possible to have assignments on certain faces of the icosahedron without disturbing the linearity of the channel distribution on the lattice network, thus no assignments are shown in the shaded areas of Fig. 1.

An important factor, which is not clear from Fig. 1 is that the density of assignments in the spherical lattice network is not uniform, although it has been derived from a uniform plane lattice network. For example, the density of assignments near the apices of the blue triangles in Fig. 1 is almost twice that at the centres of the triangles. The Appendix gives details of calculations of assignment density for spherical lattice networks.

These variations of assignment density and interference levels together with restrictions on use of assignments on certain faces of the icosahedron mean that this method of generating spherical lattice networks is not particularly attractive.

3. Use of equal-area map projections

Equal-area map projections ensure that the size of areas are correctly represented on the map. If a plane lattice network is superimposed on an equal-area map, the resulting density of assignments is uniform.

Distortion of distances caused by the map projection will result in non-uniform levels of interference. This distortion becomes significant when large parts of the Earth's surface are represented on the map. As frequency assignments are generally not required at sea, it is only necessary to minimise distortion for the land areas. Fig. 2 shows an equal-area map projection which has low distortion throughout Europe, Africa and Asia.[2] A lattice network with a nominal co-channel distance of 3,000 km has been superimposed on the map. The geographical co-ordinates of each transmitter may be calculated or read directly from the map. For each transmitter, the multiple interference level due to all of the transmitters in the network has been calculated by the spherical and plane earth methods as described in Section 2.

Fig. 2 shows the co-channel distances obtained by the spherical earth method, and the difference between co-channel interference levels calculated by the two methods for each transmitter. The variations of co-channel distance are caused by distortion due to the projection. The equal-area property of the projection ensures that, for a given transmitter, an increase in co-channel distance in one direction is balanced by a reduction in another direction. This tends to stabilise the total interference from the ring of transmitters surrounding the wanted transmitter. Fig. 2 shows that, although the actual co-channel distances are significantly different from the nominal value, the interference levels are similar to those in the equivalent plane lattice network.

The difference in interference levels calculated by the 'spherical earth' and 'plane earth' methods is an indication of the distortion of the lattice caused by fitting it to a spherical surface.

Fig. 3 shows a similar lattice network for nine channels with a nominal co-channel distance of 3,000 km. The differences in interference levels are less than 1 dB over most of Europe, Africa and Asia. It is thought that by adjusting the parameters of the map projection it would be possible to reduce the differences in interference levels at the extremities of the map.

These results indicate that equal-area map projections can be used to generate lattice networks for large areas (such as Europe, Africa and Asia) which have levels of interference comparable to those in the equivalent finite plane lattice networks.

4. Conclusions

The basic properties of lattice planning on a plane surface are uniform density of assignments and uniform interference levels. Lattice networks must be distorted to fit a spherical surface without affecting these basic properties.

Plane lattice networks can be superimposed on the plane surfaces of a polyhedron, which approximates the shape of the earth. Use of an icosahedron for this purpose results in non-uniform density of assignments and variations in interference levels between assignments.

Equal-area map projections may be used for lattice planning on a spherical surface. The resulting assignment density is uniform and, even when a large part of the Earth's surface is projected, the interference levels can be similar to those in an equivalent plane lattice network.

It is therefore recommended that spherical lattice networks should be derived from equal-area map projections, this being a method which largely retains the basic advantages of lattice planning on a plane surface.

5. References

1. EDEN, H. *and* MINNE, D. MF broadcast coverage by plane and spherical transmitter networks. EBU Review 1969 115-A, pp 109-120 June 1969.

2. LAVEN, P.A. *and* LEE, R.W. 1975. Map projections for planning of broadcasting services: use of a digital computer. BBC Research Report 1975/21.

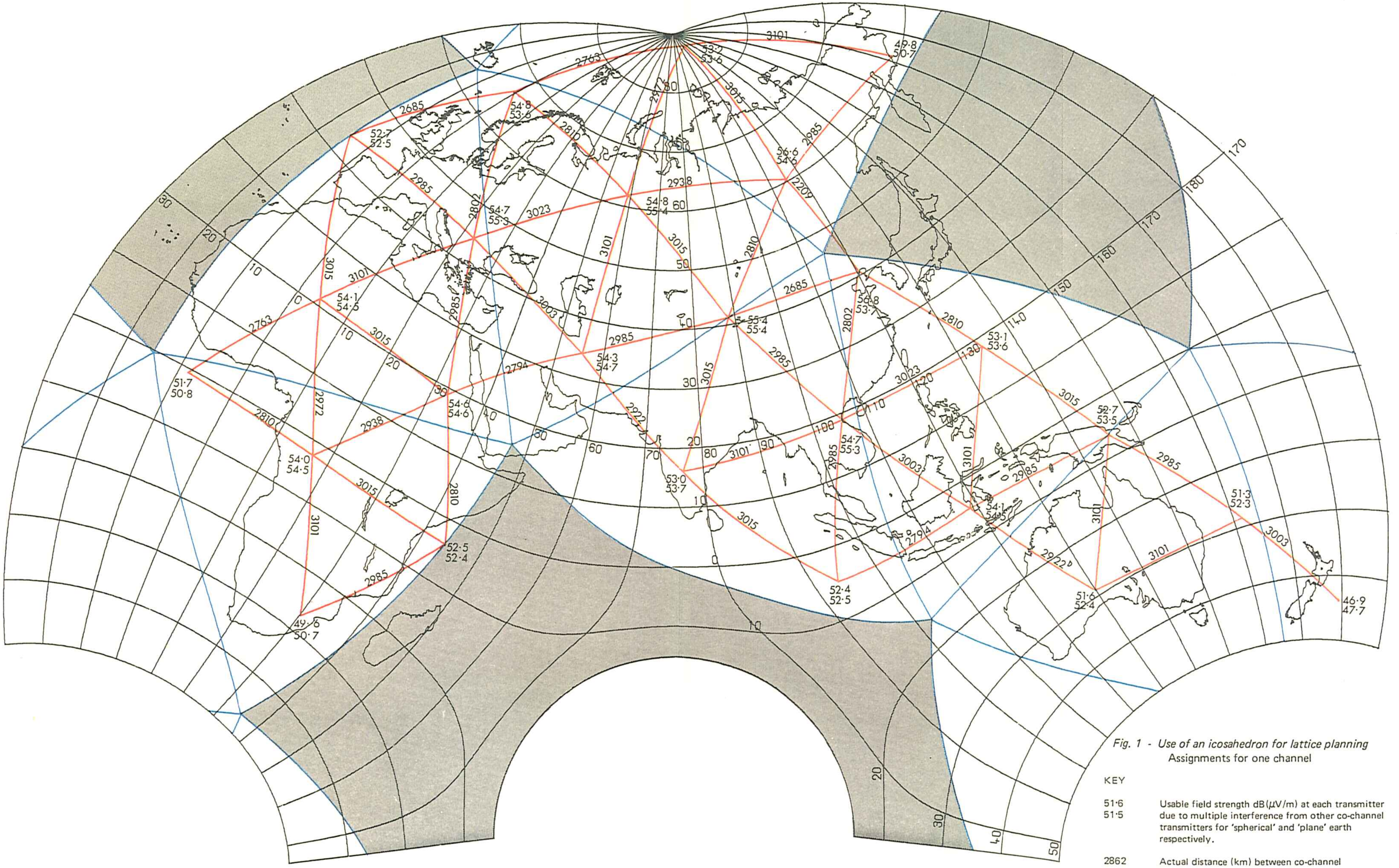

Fig. 1 - Use of an icosahedron for lattice planning
Assignments for one channel

KEY

51·6 51·5	Usable field strength dB(μV/m) at each transmitter due to multiple interference from other co-channel transmitters for 'spherical' and 'plane' earth respectively.
2862	Actual distance (km) between co-channel transmitters.

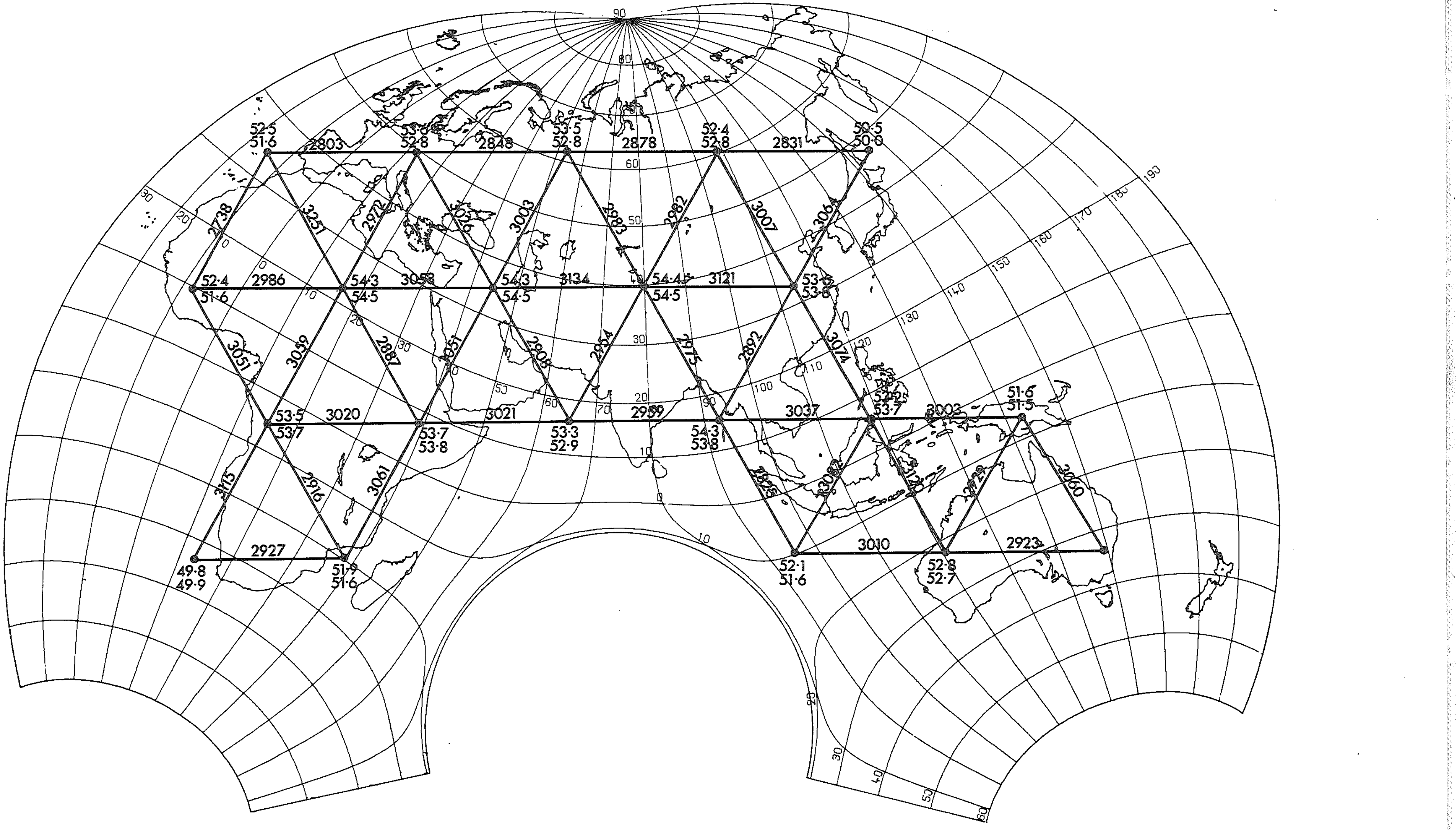

Fig. 2 - Use of map projections for lattice planning

Assignments for one channel

Nominal co-channel distance: 3000 km

KEY

51·5 51·5	Usable field strength dB(μV/m) at each transmitter due to multiple interference from other co-channel transmitters for 'spherical' and 'plane' earth respectively
2862	Actual distance (km) between co-channel transmitters

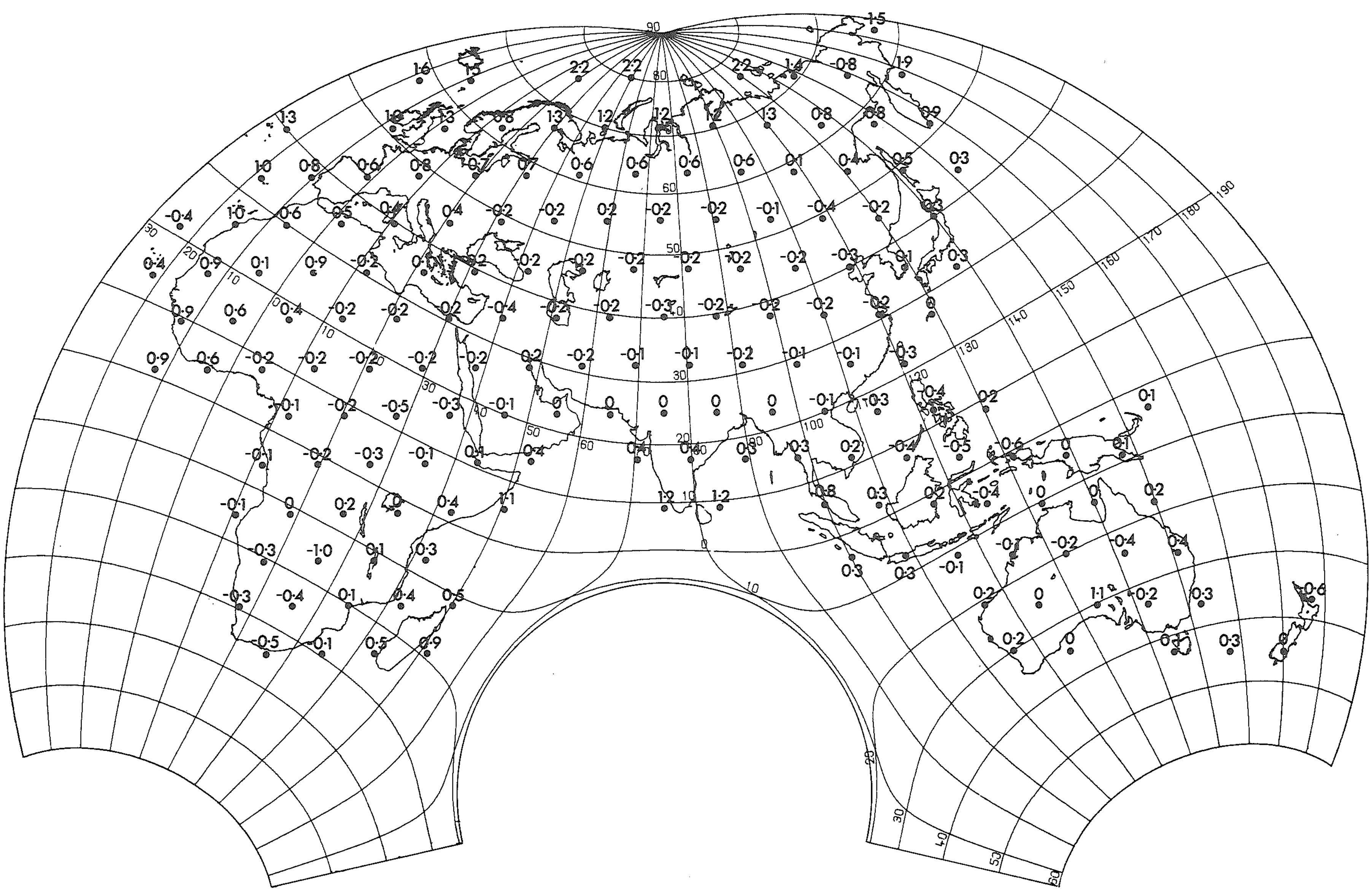

Fig. 3 - Use of map projections for lattice planning
Assignments for nine channels

Nominal co-channel distance: 3000 km

KEY

+0·3 Ratio (dB) of interference levels for each transmitter calculated for 'spherical earth' and for 'plane earth'

APPENDIX

Projection of lattice networks to a sphere from a tetrahedron, octahedron or icosahedron

Lattice networks on the plane surfaces of a tetrahedron, octahedron or icosahedron are projected on to the surface of the sphere using the gnomonic (or central) projection. This projection has the property that all straight lines on the surfaces of the polyhedron correspond to great circles on the surface of the sphere. Although this property may appear to be useful for radio planning purposes, the projection suffers from considerable distortion of scale. Projection between two surfaces without scale distortion is only possible if the surfaces are parallel. The surfaces of a sphere and a concentric polyhedron are only parallel at the points corresponding to the centres of the equilateral triangles, as shown by point P in Fig. 4.

The position of any point on the triangular face of the polyhedron may be defined by its distance s and azimuth from P. Similarly the position of the corresponding point on the sphere may be defined by its arc distance α and azimuth from P. The two azimuths are identical as the surfaces are parallel at P. The relationship between s and α for a sphere of radius r is

$$\alpha = \tan^{-1} \quad (s/r)$$

The scale distortion in a radial direction from P is

$$m_0 = \frac{d\alpha}{ds} = \cos^2\alpha$$

The scale distortion in the orthogonal direction is

$$m_{90} = \frac{\text{circumference of circle of radius } \alpha \text{ on a sphere}}{\text{circumference of circle of radius } s \text{ on a plane}} =$$

$$= \frac{2\pi r \sin\alpha}{2\pi s} = \frac{r \sin\alpha}{r \tan\alpha} = \cos\alpha$$

The distortion of the area of a small zone at distance α from P is

$$A_\alpha = m_0 \, m_{90} = \cos^3\alpha$$

The area of a zone of the surface of the sphere is smaller than that of the corresponding zone on the surface of the polyhedron by the factor A_α. If the plane lattice network on the surface of the polyhedron has a uniform density of assignments, it follows that the spherical lattice will have a higher density of assignments by the factor $1/A_\alpha$.

The important dimensions of the spherical triangles corresponding to the three types of polyhedron are shown in Table 1, which also shows the increase in assignment density at distances from point P as shown in Fig. 4. The icosahedron shows the least variation because of the smaller dimensions of triangular faces.

TABLE 1

	Dimensions of spherical triangles corresponding to triangular faces of a polyhedron (see Fig. 4)				Variation of assignment density in a spherical lattice network	
	θ	a	b	c	$\frac{1}{A_b}$	$\frac{1}{A_c}$
Tetrahedron	120°	109·47°	70·53°	54·74°	27·00	5·20
Octahedron	90°	90·00°	54·74°	35·26°	5·20	1·84
Icosahedron	72°	63·43°	37·38°	20·91°	1·99	1·23

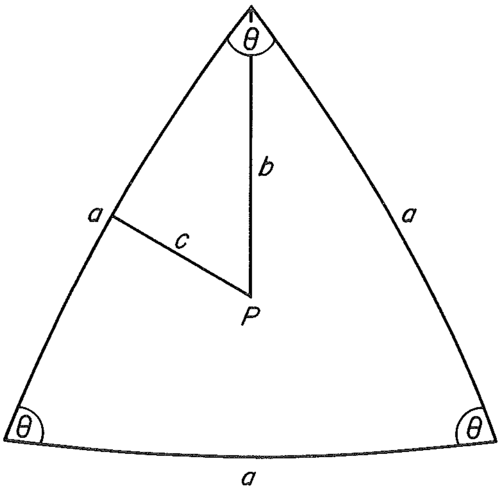

Fig. 4 - Dimensions of the spherical triangle corresponding to the plane triangular face of a tetrahedron, octahedron or icosahedron (see Table 1)

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